



**Anthony Wayne Local Schools**  
**Course of Study**  
College Math

**Anthony Wayne Local Schools Mathematics Belief Statements**

All Generals will experience an innovative and engaging curriculum with instruction that is personalized, promotes creativity and application, and provides real-world experiences that facilitate deeper learning.

***AWLS believes Mathematics instruction should:***

- identify skill gaps for individual students and work to close them
- include engaging learning activities where all learners can grow through productive struggle.
- develop strong number sense with the ability to manipulate numbers and perform mental math with an emphasis on subitizing
- provide scenarios where real world problems help to provide a path towards being future ready students.
- develop strong mathematical modeling and reasoning skills that continually build on prior knowledge.
- encourage students to be risk takers, demonstrate resilience and grit, while solving complex mathematical problems.
- encourage flexibility, creativity, and communication while working collaboratively with peers.
- include consistent and cohesive academic vocabulary through all grade-levels that is utilized by both teachers and students

**College Math Course Description:**

First semester students will review many of the topics covered in Algebra II including linear and quadratic functions, polynomial functions, inequalities, matrices, system of equations, and rational functions. Second semester students will cover vectors (2-dimensional and 3-dimensional), exponential and logarithmic functions, sequences and series, conic sections, and trigonometry. A graphing calculator is required for this course, preferably the TI 83 Plus or the TI 84.

Domain/ Conceptual Category	Standard	
Number and Quantity	N.RN.1	<b>Extend the properties of exponents to rational exponents.</b> <b>N.RN.1</b> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5(1/3)^3</math> to</i>

		<i>hold, so <math>(51/3)^3</math> must equal 5.</i>
Number and Quantity	N.RN.2	<b>Extend the properties of exponents to rational exponents.</b> <b>N.RN.2</b> Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Number and Quantity	N.Q.2	<b>Reason quantitatively and use units to solve problems.</b> <b>N.Q.2</b> Define appropriate quantities for the purpose of descriptive modeling. ★
Number and Quantity	N.CN.1	<b>Perform arithmetic operations with complex numbers.</b> <b>N.CN.1</b> Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.
Number and Quantity	N.CN.2	<b>Perform arithmetic operations with complex numbers.</b> <b>N.CN.2</b> Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
Number and Quantity	N.CN.3	<b>Perform arithmetic operations with complex numbers.</b> <b>N.CN.3</b> (+) Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.
Number and Quantity	N.CN.4	<b>Represent complex numbers and their operations on the complex plane.</b> <b>N.CN.4</b> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
Number and Quantity	N.CN.5	<b>Represent complex numbers and their operations on the complex plane.</b> <b>N.CN.5</b> (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, <math>(-1 + \sqrt{3}i)^3 = 8</math> because <math>(-1 + \sqrt{3}i)</math> has magnitude 2 and argument <math>120^\circ</math>.</i>
Number and Quantity	N.CN.6	<b>Represent complex numbers and their operations on the complex plane.</b> <b>N.CN.6</b> (+) Calculate the distance between numbers in the complex plane as the magnitude of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Number and Quantity	N.CN.7	<b>Use complex numbers in polynomial identities and equations.</b> <b>N.CN.7</b> Solve quadratic equations with real coefficients that have complex solutions.
Number and Quantity	N.CN.8	<b>Use complex numbers in polynomial identities and equations.</b> <b>N.CN.8</b> (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</i>

Number and Quantity	N.CN.9	<b>Use complex numbers in polynomial identities and equations.</b> <b>N.CN.9 (+)</b> Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Algebra	A.SSE.1	<b>Interpret the structure of expressions.</b> <b>A.SSE.1.</b> Interpret expressions that represent a quantity in terms of its context. ★ <b>a.</b> Interpret parts of an expression, such as terms, factors, and coefficients. <b>b.</b> Interpret complicated expressions by viewing one or more of their parts as a single entity.
Algebra	A.APR.1	<b>Perform arithmetic operations on polynomials.</b> <b>A.APR.1</b> Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. <b>a.</b> Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) <b>b.</b> Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)
Algebra	A.APR.4	<b>Use polynomial identities to solve problems.</b> <b>A.APR.4</b> Prove polynomial identities and use them to describe numerical relationships. <i>For example, the polynomial identity <math>(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</i>
Algebra	A.APR.7	<b>Rewrite rational expressions.</b> <b>A.APR.7 (+)</b> Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Algebra	A.CED.1	<b>Create equations that describe numbers or relationships.</b> <b>A.CED.1</b> Create equations and inequalities in one variable and use them to solve problems. <i>Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.</i> ★ <b>a.</b> Focus on applying linear and simple exponential expressions. (A1, M1) <b>b.</b> Focus on applying simple quadratic expressions. (A1, M2) <b>c.</b> Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

Algebra	A.CED.2	<p><b>Create equations that describe numbers or relationships.</b></p> <p><b>A.CED.2</b> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★</p> <p>a. Focus on applying linear and simple exponential expressions. (A1, M1)</p> <p>b. Focus on applying simple quadratic expressions. (A1, M2)</p> <p>c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3)</p>
Algebra	A.CED.3	<p><b>Create equations that describe numbers or relationships.</b></p> <p><b>A.CED.3</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i> ★</p> <p>(A1, M1)</p> <p>a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)</p>
Algebra	A.CED.4	<p><b>Create equations that describe numbers or relationships.</b></p> <p><b>A.CED.4</b> Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★</p> <p>a. Focus on formulas in which the variable of interest is linear or square. <i>For example, rearrange Ohm's law <math>V=IR</math> to highlight resistance <math>R</math>, or rearrange the formula for the area of a circle <math>A=(\pi)r^2</math> to highlight radius <math>r</math>.</i> (A1)</p> <p>b. Focus on formulas in which the variable of interest is linear. <i>For example, rearrange Ohm's law <math>V=IR</math> to highlight resistance <math>R</math>.</i> (M1)</p> <p>c. Focus on formulas in which the variable of interest is linear or square. <i>For example, rearrange the formula for the area of a circle <math>A=(\pi)r^2</math> to highlight radius <math>r</math>.</i> (M2)</p> <p>d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)</p>
Algebra	A.REI.2	<p><b>Understand solving equations as a process of reasoning and explain the reasoning.</b></p> <p><b>A.REI.2</b> Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p>

Algebra	A.REI.4	<p><b>Solve equations and inequalities in one variable.</b></p> <p><b>A.REI.4</b> Solve quadratic equations in one variable.</p> <p><b>a.</b> Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions.</p> <p><b>b.</b> Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for <math>x^2 = 49</math>; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.</p> <p>(+) <b>c.</b> Derive the quadratic formula using the method of completing the square.</p>
Algebra	A.REI.11	<p><b>Represent and solve equations and inequalities graphically.</b></p> <p><b>A.REI.11</b> Explain why the <math>x</math>-coordinates of the points where the graphs of the equation <math>y=f(x)</math> and <math>y=g(x)</math> intersect are the solutions of the equation <math>f(x)=g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.</p>
Functions	F.IF.1	<p><b>Understand the concept of a function, and use function notation.</b></p> <p><b>F.IF.1</b> Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p>
Functions	F.IF.3	<p><b>Understand the concept of a function, and use function notation.</b></p> <p><b>F.IF.3</b> Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p>
Functions	F.IF.4	<p><b>Interpret functions that arise in applications in terms of the context.</b></p> <p><b>F.IF.4</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> ★ (A2, M3)</p> <p><b>a.</b> Focus on linear and exponential functions. (M1)</p> <p><b>b.</b> Focus on linear, quadratic, and exponential functions. (A1, M2)</p>

Functions	F.IF.5	<p><b>Interpret functions that arise in applications in terms of the context.</b></p> <p><b>F.IF.5</b> Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★</p> <p>a. Focus on linear and exponential functions. (M1)</p> <p>b. Focus on linear, quadratic, and exponential functions. (A1, M2)</p> <p>c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)</p>
Functions	F.BF.3	<p><b>Build new functions from existing functions.</b></p> <p><b>F.BF.3</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x)+k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x+k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i> (A2, M3)</p> <p>a. Focus on transformations of graphs of quadratic functions, except for <math>f(kx)</math>. (A1, M2)</p>
Functions	F.LE.5	<p><b>Interpret expressions for functions in terms of the situation they model.</b></p> <p><b>F.LE.5</b> Interpret the parameters in a linear or exponential function in terms of a context. ★</p>
Geometry	G.GPE.1	<p><b>Translate between the geometric description and the equation for a conic section.</b></p> <p><b>G.GPE.1</b> Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</p>
Statistics and Probability	S.ID.6	<p><b>Summarize, represent, and interpret data on two categories and quantitative variables</b></p> <p><b>S.ID.6</b> Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i> (A2, M3)</p> <p>b. Informally assess the fit of a function by discussing residuals. (A2, M3)</p>

		c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)
Number and Quantity	N.Q.1	<b>Reason quantitatively and use units to solve problems.</b> <b>N.Q.1</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★
Number and Quantity	N.Q.2	<b>Reason quantitatively and use units to solve problems.</b> <b>N.Q.2</b> Define appropriate quantities for the purpose of descriptive modeling.★
Algebra	A.SSE.2	<b>Interpret the structure of expressions.</b> <b>A.SSE.2</b> Use the structure of an expression to identify ways to rewrite it. <i>For example, to factor <math>3x(x - 5) + 2(x - 5)</math>, students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to <math>(3x+2)(x - 5)</math>; or see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>
Algebra	A.SSE.3	<b>Write expressions in equivalent forms to solve problems.</b> <b>A.SSE.3</b> Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ <b>a.</b> Factor a quadratic expression to reveal the zeros of the function it defines. <b>b.</b> Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. <b>c.</b> Use the properties of exponents to transform expressions for exponential functions. <i>For example, <math>8t</math> can be written as <math>2^{3t}</math>.</i>
Algebra	A.APR.2	<b>Understand the relationship between zeros and factors of polynomials.</b> <b>A.APR.2</b> Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ . In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
Algebra	A.APR.3	<b>Understand the relationship between zeros and factors of polynomials.</b> <b>A.APR.3</b> Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
Algebra	A.APR.6	<b>Rewrite rational expressions.</b> <b>A.APR.6</b> Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

Algebra	A.REI.5	<p><b>Solve systems of equations.</b></p> <p><b>A.REI.5</b> Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p>
Algebra	A.REI.6	<p><b>Solve systems of equations.</b></p> <p><b>A.REI.6</b> Solve systems of linear equations algebraically and graphically.</p> <p>a. Limit to pairs of linear equations in two variables. (A1, M1)</p> <p>b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)</p>
Algebra	A.REI.7	<p><b>Solve systems of equations.</b></p> <p><b>A.REI.7</b> Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i></p>
Algebra	A.REI.8	<p><b>Solve systems of equations.</b></p> <p><b>A.REI.8 (+)</b> Represent a system of linear equations as a single matrix equation in a vector variable.</p>
Functions	F.IF.8	<p><b>Analyze functions using different representations.</b></p> <p><b>F.IF.8</b> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)</p> <p>i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2)</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change <math>G</math> in functions such as <math>y = (1.02)t</math>, and <math>y = (0.97)t</math> and classify them as representing exponential growth or decay.</i> (A2, M3)</p> <p>i. Focus on exponential functions evaluated at integer inputs. (A1, M2)</p>
Functions	F.BF.4	<p><b>Build new functions from existing functions.</b></p> <p><b>F.BF.4</b> Find inverse functions.</p> <p>a. Informally determine the input of a function when the output is known. (A1, M1)</p> <p>b. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)</p> <p>c. (+) Verify by composition that one function is the inverse of another. (A2, M3)</p> <p>d. (+) Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)</p>



		e. (+) Produce an invertible function from a non-invertible function by restricting the domain.
Functions	F.LE.4	<b>Construct and compare linear, quadratic, and exponential models, and solve problems.</b> <b>F.LE.4</b> For exponential models, express as a logarithm the solution to $abct = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology. ★
Number and Quantity	N.VM.6	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.6</b> (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
Number and Quantity	N.VM.7	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.7</b> (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
Number and Quantity	N.VM.8	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.8</b> (+) Add, subtract, and multiply matrices of appropriate dimensions.
Number and Quantity	N.VM.9	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.9</b> (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
Number and Quantity	N.VM.10	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.10</b> (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
Number and Quantity	N.VM.11	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.11</b> (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
Number and Quantity	N.VM.12	<b>Perform operations on matrices, and use matrices in applications.</b> <b>N.VM.12</b> (+) Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
Algebra	A.SSE.4	<b>Write expressions in equivalent forms to solve problems.</b> <b>A.SSE.4</b> (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. <i>For example, calculate mortgage payments.</i> ★

Algebra	A.APR.5	<p><b>Use polynomial identities to solve problems.</b></p> <p><b>A.APR.5</b> (+) Know and apply the Binomial Theorem for the expansion of <math>(x + y)^n</math> in powers of <math>x</math> and <math>y</math> for a positive integer <math>n</math>, where <math>x</math> and <math>y</math> are any numbers. <i>For example by using coefficients determined for by Pascal's Triangle.</i> The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.</p>
Functions	F.BF.1	<p><b>Build a function that models a relationship between two quantities.</b></p> <p><b>F.BF.1</b> Write a function that describes a relationship between two quantities. ★</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>i. Focus on linear and exponential functions. (A1, M1)</p> <p>ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)</p> <p>b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i> (A2, M3)</p> <p>c. (+) Compose functions. <i>For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time .</i></p>
Functions	F.BF.2	<p><b>Build a function that models a relationship between two quantities.</b></p> <p><b>F.BF.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★</p>
Functions	F.LE.2	<p><b>Construct and compare linear, quadratic, and exponential models, and solve problems.</b></p> <p><b>F.LE.2</b> Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★</p>
Geometry	G.GPE.2	<p><b>Translate between the geometric description and the equation for a conic section.</b></p> <p><b>G.GPE.2</b> (+) Derive the equation of a parabola given a focus and directrix.</p>
Geometry	G.GPE.3	<p><b>Translate between the geometric description and the equation for a conic section.</b></p> <p><b>G.GPE.3</b> (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.</p>
Functions	F.TF.1	<p><b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p><b>F.TF.1</b> Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p>

Functions	F.TF.2	<p><b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p><b>F.TF.2</b> Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p>
Functions	F.TF.3	<p><b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p><b>F.TF.3 (+)</b> Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosine, and tangent for <math>\pi-x</math>, <math>\pi+x</math>, and <math>2\pi-x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p>
Functions	F.TF.4	<p><b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p><b>F.TF.4 (+)</b> Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p>
Functions	F.TF.5	<p><b>Model periodic phenomena with trigonometric functions.</b></p> <p><b>F.TF.5</b> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★</p>
Number and Quantity	N.VM.1	<p><b>Represent and model with vector quantities.</b></p> <p><b>N.VM.1 (+)</b> Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes, e.g., <math>\mathbf{v}</math>, <math> \mathbf{v} </math>, <math>\ \mathbf{v}\ </math>, <math>v</math>.</p>
Number and Quantity	N.VM.2	<p><b>Represent and model with vector quantities.</b></p> <p><b>N.VM.2 (+)</b> Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p>
Number and Quantity	N.VM.3	<p><b>Represent and model with vector quantities.</b></p> <p><b>N.VM.3 (+)</b> Solve problems involving velocity and other quantities that can be represented by vectors.</p>
Number and Quantity	N.VM.4	<p><b>Perform operations on vectors.</b></p> <p><b>N.VM.4 (+)</b> Add and subtract vectors.</p> <p><b>a.</b> Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p><b>b.</b> Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p><b>c.</b> Understand vector subtraction <math>\mathbf{v} - \mathbf{w}</math> as <math>\mathbf{v} + (-\mathbf{w})</math>, where <math>-\mathbf{w}</math> is the additive inverse of <math>\mathbf{w}</math>, with the same magnitude as <math>\mathbf{w}</math> and pointing in the opposite direction. Represent vector subtraction graphically by</p>

		connecting the tips in the appropriate order, and perform vector subtraction component-wise.
Number and Quantity	N.VM.5	<b>Perform operations on vectors.</b> <b>N.VM.5 (+)</b> Multiply a vector by a scalar. <b>a.</b> Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ . <b>b.</b> Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\  =  c v$ . Compute the direction of $c\mathbf{v}$ knowing that when $ c v \neq 0$ , the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$ ) or against $\mathbf{v}$ (for $c < 0$ ).